A Method to Solve Inverse Kinematics of Redundant Slave Arm in The Master-Slave System with Different Degrees of Freedom

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Abstract

A master-slave manipulator with different degrees of freedom has some advantages, such as operational performance. However, it is difficult to determine the angles of a redundant slave manipulator in such a system. In this paper, we determine them by measuring the operator's posture with an acceleration sensor to follow the operator's motion as exactly as possible.

1. Introduction

There have been extensive research efforts on master-slave robot systems for teleoperation. Since most of them are for disaster relief and recovery activities, slave robots are composed of heavy mechanism, preventing the user's smooth operation. On the other hand, we have proposed master-slave system that works in our daily life [1]. In this system, slave robot can not only perform precise handiworks, but also communicate with surrounding people with gestures. We call this concept “mutual telexistence”, because the operator virtually exists in the remote place not only from the operator’s view, but also from surrounding people.

The specification to satisfy these two demands (precise handiwork + communication with gestures) is as follows. First, the slave arm must have 7-DOF in order to express an operator's gesture. Just like human arm, the first and last three joints must intersect at single points, which are shoulder and wrist, while the remaining one joint represents elbow (Fig.1).

The second specification is master arm. When the slave arm has redundant degree of freedom, common bilateral control methods are the symmetric position servo type and the force reflecting servo type between corresponding joints by using exoskeleton type 7-DOF master arm. However, to restrain an operator with active master mechanism is not only dangerous but also leads to restriction of an operator’s motion, which makes the range of operation narrower because of the mechanism complicated by components such as motor and sensor. Therefore, as for master arm, it is desirable to have 6-DOF, not 7-DOF, and perform force feedback control adequately without restraining an operator by mechanism except for his wrist. In our prototype master manipulator, the last three joints intersect at a single point using gimbal ring structure (Fig.2).
2. Master-slave manipulator with different degrees of freedom and its inverse kinematics

As our master arm has 6-DOF and slave arm has 7-DOF, simple symmetric servo between corresponding joints cannot be applied. Remember our two demands: precise handiwork and communication with gesture. From the first requirement, the position and orientation of slave arm’s wrist must coincide with master’s. 6-DOF is used for this purpose. From the second requirement, slave’s posture must match as close as possible as operator. Remaining 1-DOF is used for this purpose. Note that the conventional method based on the pseudo inverse of Jacobian matrix is not appropriate, because it does not satisfy the second requirement.

There are some possible ways to measure an operator’s posture such as using markers and a camera, which is adopted in general motion capture system [3]. However, the optical method has two problems: time-delay and occlusion. Therefore, we take another simpler method mentioned below to avoid these problems. By this measurement, master side has 7-DOF for position and orientation, while it has 6-DOF for force feedback (Fig.3).

The method to solve inverse kinematics of slave arm is as follows. The origin (0, 0, 0) is coincident with the shoulder position and \( L_1, L_2, L_3 \) are the length of the upper arm, lower arm, and the distance from the goal position to the shoulder of the slave arm. First, the goal position \( P \) and orientation \( R \) (rotation matrix) of slave arm’s wrist are determined by forward kinematics of master arm. Then the joint angle of the elbow \( \theta_4 \) is determined geometrically (Fig.5).
In Fig. 5, \( \mathbf{n} \) is a unit vector from the shoulder to the wrist. Additionally, \( \mathbf{u} \) and \( \mathbf{v} \) are unit vectors which form a local coordinate system for the plane containing the circle which elbow traces. \( \mathbf{u} \) is determined arbitrarily to correspond to \( \phi = 0 \). Then \( \mathbf{v} \) is set as \( \mathbf{v} = \mathbf{n} \times \mathbf{u} \).

Similarly to \( \theta_4 \), \( \alpha \) is determined and the center of the circle \( \mathbf{c} \) which elbow traces and its radius \( r \) can be computed as

\[
\cos \alpha = \frac{L_1^2 + L_2^2 - L_3^2}{2L_1L_2},
\]

\[
\mathbf{c} = L_1 \cos \alpha \mathbf{a}, \quad r = L_1 \sin \alpha.
\]

If we set \( \mathbf{u} \) the direction that the output of the acceleration sensor \( a \) is equal to the gravitational acceleration \( g \)

\[
\mathbf{u} = \frac{(n_z, 0, -n_z)^T}{\sqrt{n_x^2 + n_y^2}},
\]

the swivel angle \( \phi \) is obtained as

\[
\sin \phi = \frac{a}{g}.
\]

Then the elbow position \( \mathbf{E}(\phi) \) is given by

\[
\mathbf{E}(\phi) = \mathbf{c} + r(\cos \phi \mathbf{a} + \sin \phi \mathbf{v}).
\]

From the relation between this elbow position \( \mathbf{E}(\phi) \) and wrist position \( \mathbf{P} \), three joint angles of the shoulder \( \theta_1 \sim \theta_3 \) are determined. Next, from the orientation \( \mathbf{R} \) of the wrist, three joint angles of the wrist \( \theta_5 \sim \theta_7 \) are determined.

As described above, we can solve inverse kinematics of the redundant slave arm which satisfies the wrist’s position and orientation obtained by the master arm and elbow position computed from an acceleration sensor’s output.

3. Experiment

3.1. Procedure

We constructed a master-slave system consisted of 7-DOF slave arm, 6-DOF master arm, and an acceleration sensor attached on an operator’s upper arm. We examined the accuracy of slave arm’s elbow position by comparing it with that of the operator. The elbow position of the operator is measured by a link type measurement instrument (ADL-1, Shooting Star Technology). The joint angles of the slave arm are determined by the acceleration sensor using the above algorithm. In the first half period (0~ 30 seconds), the operator does not move except for his elbow and in the latter half (30~ 60 seconds), he moved his arm freely.

3.2. Result

The result is shown in Fig. 6. In the first half, the gap of the elbow position between an operator and slave arm is within 5[cm], while in the second half, it is within 10[cm]. At the same time, we observe quite little time delay. The mismatch of the elbow position is caused by the following two reasons. One is the difference of arm length between the operator and slave arm. The other is the wrong value of the acceleration sensor. We assumed that the sensor measures gravity direction, but it also measures the acceleration of the arm itself. The easiest way to solve the latter problem is to apply low-pass filter, but we must be careful about the trade-off between strength of the filter and time-delay. Another solution is to use a gyro sensor together with the acceleration sensor to compensate for the acceleration of an operator’s motion. However, as our purpose of measuring the elbow position is for communication with gesture, the accuracy got by only an acceleration sensor is enough.

4. Conclusion

In this paper, we introduced a method to determine the joint angles of redundant slave arm by the measurement
of an operator’s posture. Especially we examined the case in which master arm has 6-DOF and slave arm has 7-DOF. As our purpose includes communication with gesture, the conventional method based on pseudo inverse of Jacobian matrix was not adequate. We used an acceleration sensor attached on the operator’s upper arm to measure redundant DOF. By experiment, we showed enough accuracy of the slave elbow position computed from the output of the acceleration sensor.

![Fig.6 Elbow position of x, y, and z-axis](image)

5. References


